Completing the Square

Say you are asked to solve the equation:

$$\mathbf{x^2 + 6x + 2 = 0}$$

We cannot use any of the techniques in factorization to solve for x. In this situation, we use the technique called completing the square. This makes the quadratic equation into a perfect square trinomial,

i.e. the form $a^2 + 2ab + b^2 = (a + b)^2$.

NOTE: This technique is valid only when 1 is the coefficient of x^2.

Here are the steps used to complete the square

Step 1. Move the constant term to the right:

$$x^2 + 6x = -2$$

Step 2. Add the square of half the coefficient of x to both sides. In this case, add the square of half of 6 i.e. add the square of 3.

$$x^2 + 6x + 9 = -2 + 9$$

The left-hand side is now the perfect square of (x + 3).

Note that the 3 in (x+3) comes from half of the coefficient of x or half of 6

$$(x + 3)^2 = 7$$

We know that if $a^2 = b$, $a = \pm b$

Therefore, $x + 3 = \pm\sqrt{7}$

$$x = -3 \pm \sqrt{7}$$

Here is another example, Solve: $x^2 - 2x - 2 = 0$

Step 1. Move the constant term to the right:

$$x^2 - 2x = 2$$

Step 2. Add the square of half the coefficient of x to both sides. In this case, add the square of half of -2 i.e. add the square of -1.

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$x^2 - 2x + 1 = 2 + 1$

The left-hand side is now the perfect square of (x - 1).

$(x - 1)^2 = 3$

We know that if $a^2 = b, a = \pm\sqrt{b}$

Therefore, $x - 1 = \pm\sqrt{3}$

Note that the -1 in (x-1) comes from half of the coefficient of x or half of -2

$$x = 1 \pm \sqrt{3}$$

STRATEGY USED TO COMPLETE THE SQUARE

for any equation of the form $x^2 + ax + b = 0$

1. Move b to the right side such that $x^2 + ax = -b$
2. Add $\left(\frac{a}{2}\right)^2$ to both sides such that $x^2 + ax + \left(\frac{a}{2}\right)^2 = -b + \left(\frac{a}{2}\right)^2$
3. We can see that $x^2 + ax + \left(\frac{a}{2}\right)^2$ is now a perfect square of $\left(x + \frac{a}{2}\right)^2$
4. The square is now completed and simple algebraic calculations will lead to the values of x.

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Completing the Square

Try some practice problems:

1. $x^2 + 5x + 2 = 0$
2. $x(2x + 3) = 6$
3. $x^2 + \left(\frac{4}{3}\right)x = 0$
4. $t^2 + 2t = 15$