The Derivative

1. By definition: \( f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \)

Example: find \( f'(x) \) of \( f(x) = x^2 + 2x \)

Solution:
\[
    f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\
    = \lim_{h \to 0} \frac{(x + h)^2 + 2(x + h) - (x^2 + 2x)}{h} \\
    = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + 2x + 2h - x^2 - 2x}{h} \\
    = \lim_{h \to 0} \frac{2hx + h^2 + 2h}{h} \\
    = \lim_{h \to 0} (2x + h + 2) \\
    = 2x + 2
\]

Questions: Find the derivative of each of the following functions.

1. \( f(x) = 2x^2 \)
2. \( f(x) = 3x^2 + 2x - 1 \)
3. \( f(x) = \frac{1}{x} \)

*** \( \frac{dy}{dx} = \frac{df(x)}{dx} = \frac{d}{dx}f(x) \) and \( f'(x) \) all represent the derivative. They are all the same, just in different forms.
The Derivative

By Simple Rule:

Derivative of a constant number is 0. \( f(x) = A \rightarrow f'(x) = 0 \)

Compare the following two equations, notice that the additive constants disappear and multiplicative constants are preserved.

\[
y = A + f(x) \rightarrow y' = f'(x)
\]

\[
y = Af(x) \rightarrow y' = Af'(x)
\]

\( f(x) = x^a \rightarrow f'(x) = ax^{a-1} \quad (a \text{ can be any constant number}) \)

Example: Find the derivative \( f(x) = 3x^3 - 6x^2 + 2x - 29 \)

Solution: \( f'(x) = 3*3x^{3-1} - 6*2x^{2-1} + 2x^{1-1} = 9x^2 - 12x^1 + 2x^0 = 9x^2 - 12x + 2 \)

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