

# The Derivative

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**1. By definition:**  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

Example: find  $f'(x)$  of  $f(x) = x^2 + 2x$

Solution:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{((x+h)^2 + 2(x+h)) - (x^2 + 2x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 + 2x + 2h - x^2 - 2x}{h} \\ &= \lim_{h \rightarrow 0} \frac{2hx + h^2 + 2h}{h} \\ &= \lim_{h \rightarrow 0} 2x + h + 2 \\ &= 2x + 2 \end{aligned}$$

**Questions:** Find the derivative of each of the following functions.

1.  $f(x) = 2x^2$
2.  $f(x) = 3x^2 + 2x - 1$
3.  $f(x) = \frac{1}{x}$

\*\*\* $\frac{dy}{dx} = dy/dx$ ,  $\frac{df(x)}{dx} = df(x)/dx$ ,  $\frac{d}{dx}f(x)$  and  $f'(x)$  all represent the derivative. They are all the same, just in different forms.

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## By Simple Rule:

Derivative of a constant number is 0.  $f(x) = A \rightarrow f'(x) = 0$

Compare the following two equations, notice that the additive constants disappear and multiplicative constants are preserved.

$$y = A + f(x) \rightarrow y' = f'(x)$$

$$y = Af(x) \rightarrow y' = Af'(x)$$

$$f(x) = x^a \rightarrow f'(x) = ax^{a-1} \quad (a \text{ can be any constant number})$$

**Example:** Find the derivative  $f(x) = 3x^3 - 6x^2 + 2x - 29$

$$\text{Solution: } f'(x) = 3 \cdot 3x^{3-1} - 6 \cdot 2x^{2-1} + 2x^{1-1} = 9x^2 - 12x^1 + 2x^0 = 9x^2 - 12x + 2$$

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