Factoring Polynomials

A polynomial is a sum or subtraction of monomials. i.e. $10x^2 = 15x$ is a polynomial and so is $x^3 + 3x^2 - 5x + 9$.

In any factorization problem, the first thing to look at is the *greatest common factor*.

Factoring out the Greatest Common Factor (GCF)

* $18x^3 + 27x^2$

In this polynomial, 9 is the greatest integer that divides 18 and 27 x^2 is the greatest expression that divides x^3 and x^2.

$= 9x^2(2x) + 9x^2(3)$

$= 9x^2(2x + 3)$

Some polynomials may have a GCF of 1, but appropriate grouping may lead to possible factorization.

Factoring by Grouping

* $x^3 + 4x^2 + 3x + 12$

$= [x^3 + 4x^2] + [3x + 12]$

Common factor in $x^3 + 4x^2$ is x^2 and common factor in $3x + 12$ is 3. These can be factored as:
$= x^2 (x + 4) + 3(x+4)$

$= (x+4)(x^2 + 3)$

Factoring Trinomials

A strategy for factoring $ax^2 + bx + c$

1. Find 2 numbers whose product is *ac* and whose sum gives *b*. Say the numbers are *u* and *v*.
2. Re-write the trinomial such as $ax^2 + ux + vx + c$
3. Use factoring by grouping to find the factors

Here is an example:

* $x^2 + 3x - 18$

We need to find two numbers such that their product is -18 and their sum is 3. Let's find the factors of -18.

Factors of -18	18, -1	-18, 1	9, -2	-9, 2	**6, -3**	-6,3
Sum of factors	17	-17	7	-7	**3**	-3

Our desired numbers are 6 and -3.

We will now re-write our trinomial: $x^2 + 6x - 18$

$= [x^2 + 6x] - [3x + 18]$

$= x(x + 6) -3(x+6)$

$= (x - 3)(x+6)$

Let's look at another example whose leading coefficient is not 1:

* $8x^2 - 10x - 3$

We need to find two numbers such that their product is -24 and their sum is -10. Let's find the factors of -24.

Factors of -24	24, -1	-24, 1	12, -2	**-12, 2**	8, -3	-8, 3	6, -4	-6, 4
Sum of Factors	23	-23	10	**-10**	5	-5	2	-2

Our desire numbers are -12 and 2.

We will now re-write our trinomial: $8x^2 - 12x + 2x - 3$

$= [8x^2 -12x] + [2x - 3]$

$= 4x(2x - 3) + 1(2x-3)$

$= (4x + 1)(2x -3)$

Factoring Polynomials

A Strategy for Factoring a Polynomial

1. If there is a common factor, factor out the GCF.
2. Determine the number of terms in the polynomial and try factoring as follows
   a. If there are **two** terms, then can the binomial be factored by any of the special forms?
      i.e. Difference of two squares $a^2 - b^2 = (a + b)(a - b)$
      Sum of two cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
      Difference of two cubes $a^3 + b^3 = (a - b)(a^2 + ab + b^2)$
   b. If there are three terms, then is the trinomial a perfect square trinomial?
      i.e. $a^2 + 2ab + b^2 = (a + b)^2$
      and $a^2 - 2ab + b^2 = (a - b)^2$
      If not, try the method shown earlier for factorization of trinomials.
   c. If there are four or more terms, try factoring by grouping.
3. Check to see if any factors with more than one term in the factored polynomial can be factored further. If so, factor completely.

Let's look at another example:

* $2x^3 + 8x^2 + 8x$

Step 1: The GCF is 2x, therefore the polynomial can be written as: $2x(x^2 + 4x + 4)$

Step 2: Now we have to factor a polynomial with three terms and it looks like a perfect square trinomial. Therefore, $(x^2 + 4x + 4)$ can be factored as $(x + 2)^2$

Step 3: Check to see if factors can be factored further. If not, you have reached the final answer i.e. $2x^3 + 8x^2 + 8x = 2x(x + 2)^2$