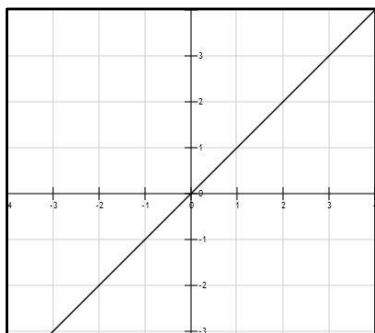
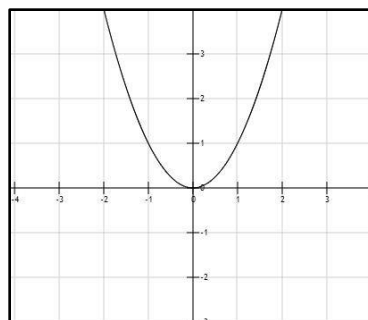


Graphing Standard Function & Transformations

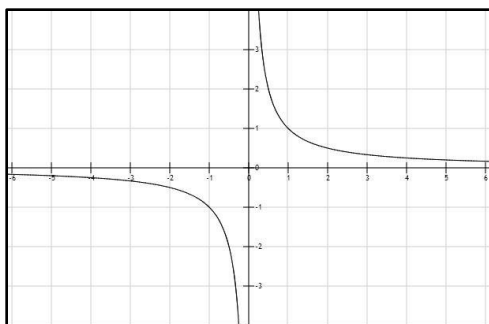
A few standard graphs



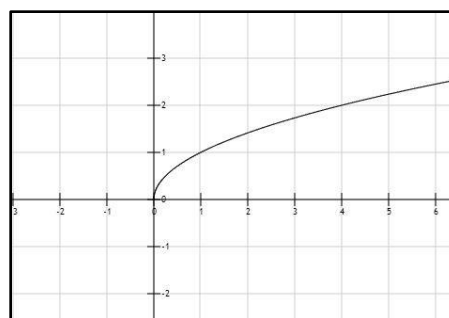
$$f(x) = x$$



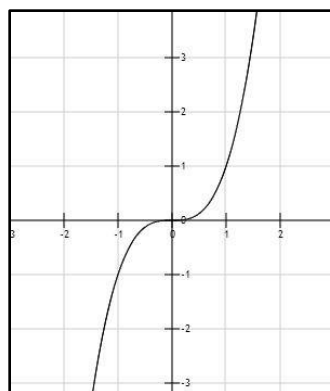
$$f(x) = x^2$$



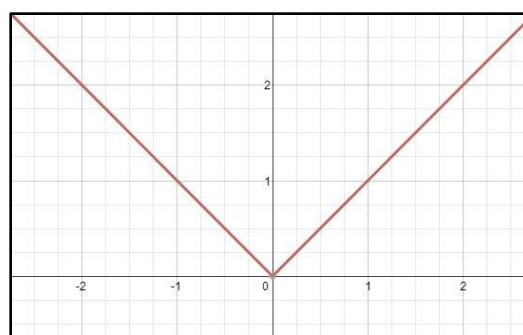
$$f(x) = \frac{1}{x}$$



$$f(x) = x^{1/2}$$



$$f(x) = x^3$$



$$f(x) = |x|$$

Graphing Standard Function & Transformations

The rules below take these standard plots and shift them horizontally/vertically

Vertical Shifts

Let f be the function and c a positive real number.

- The graph of $y = f(x) + c$ is the graph of $y = f(x)$ shifted c units vertically upwards.
- The graph of $y = f(x) - c$ is the graph of $y = f(x)$ shifted c units vertically downwards.

$$g(x) = x^2 + 2 = f(x) + 2$$

The graph of

Shifts the graph of f up 2 units

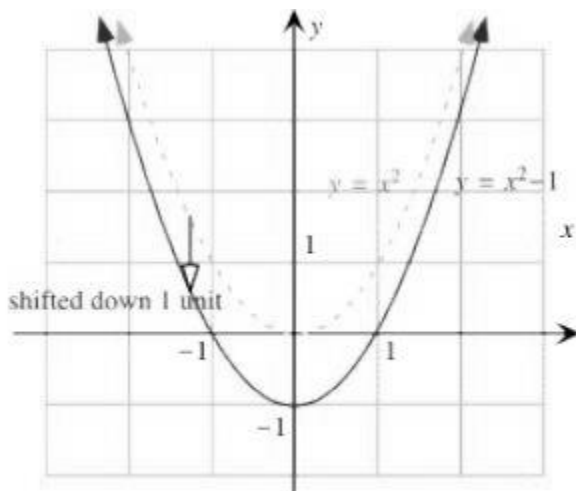
$$h(x) = x^2 - 3 = f(x) - 3$$

The graph

Shifts the graph of f down 3 units

Look for the **positive** and **negative** sign. **Positive** sign makes the graph move upwards and the **negative** sign makes it move *downwards*

Here is a picture of the graph of $g(x) = x^2 - 1$. It is obtained from the graph of $f(x) = x^2$ by shifting it down 1 unit.



Graphing Standard Function & Transformations

Horizontal Shifts

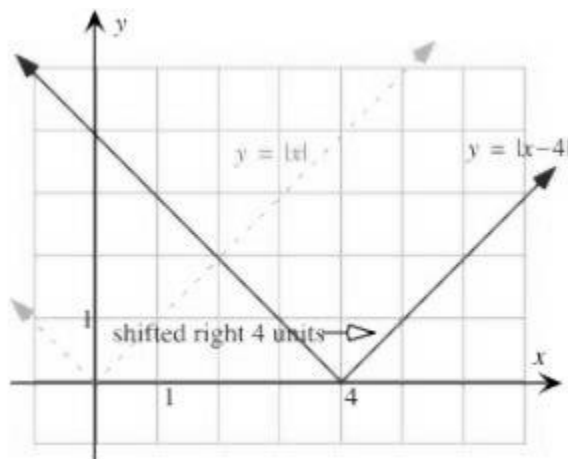
Let f be a function and c a positive real number.

- The graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted to the left c units.
- The graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted to the right c units.

$$g(x) = (x-3)^2 = f(x-3)$$

$$h(x) = (x + 2)^2 = f(x+2)$$

Here is a picture of the graph of $g(x) = |x-4|$. It is obtained from the graph of $f(x) = |x|$ by shifting it to the right 4 units.



Horizontal/ Vertical Scaling

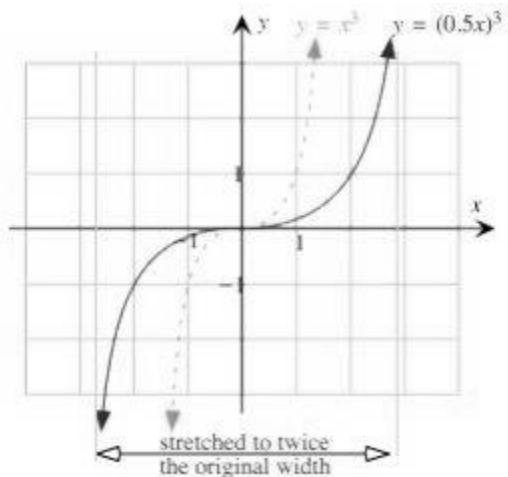
Horizontal Scaling

Let $g(x) = f(cx)$ where c is a positive real number.

Graphing Standard Function & Transformations

- If $c > 1$, the graph of g is the graph of f , stretched in the y -direction by a factor of c .
- If $0 < c < 1$, the graph of g is the graph of f , compressed in the y -direction by a factor of $1/c$.

Here is a picture of the graph of $g(x) = (0.5x)^3$. Since $c = 0.5 < 1$, the graph is obtained from that of $f(x) = x^3$ by stretching it in the x -direction by a factor of $1/c = 2$.



1, the graph of g is the compressed in the x -direction by a factor of c .

< 1 , then the graph is x -direction by a factor

picture of the graph of $g(x) = (0.5x)^3$. Since $c = 0.5 < 1$, the graph is obtained from that of $f(x) = x^3$ by stretching it in the x -direction by a factor of $1/c = 2$.

Vertical Scaling

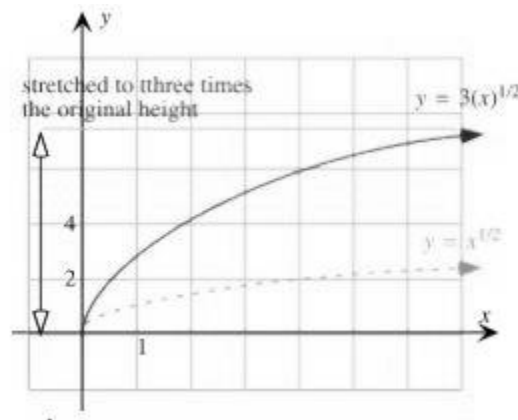
Let $g(x) = cf(x)$ here c is a positive real number.

- If $c > 1$, the graph of g is the graph of f , stretched in the y -direction by a factor of c .

Graphing Standard Function & Transformations

- If $0 < c < 1$, then the graph is compressed in the y-direction by a factor of $1/c$.

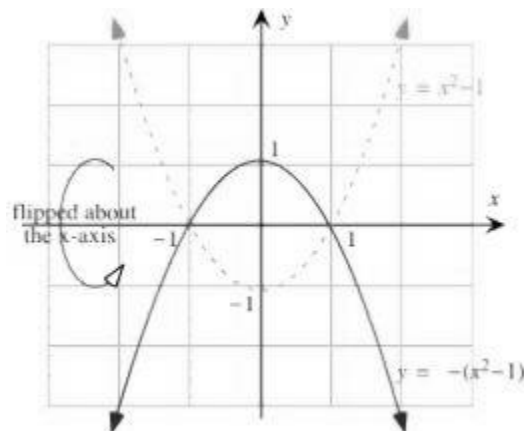
Here is a picture of the graph of $g(x) = 3(x)^{1/2}$. Since $c = 3 > 1$, the graph is obtained from that of $f(x) = x^{1/2}$ by stretching it in the y-direction by a factor of $c = 3$.



Reflection about the x axis

The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected about the x-axis.

Here is a picture of the graph of $g(x) = -(x^2 - 1)$. It is obtained from the graph of $f(x) = x^2 - 1$ by reflecting it in the x-axis.

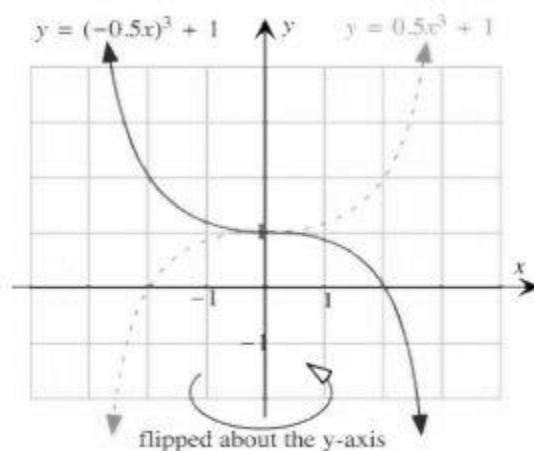


Graphing Standard Function & Transformations

Reflection about the y axis

The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected about the y-axis.

Here is a picture of the graph of $g(x) = (0.5x)^3 + 1$. It is obtained from the graph of $f(x) = 0.5x^3 + 1$ by reflecting it in the y-axis.



Summary of Transformations

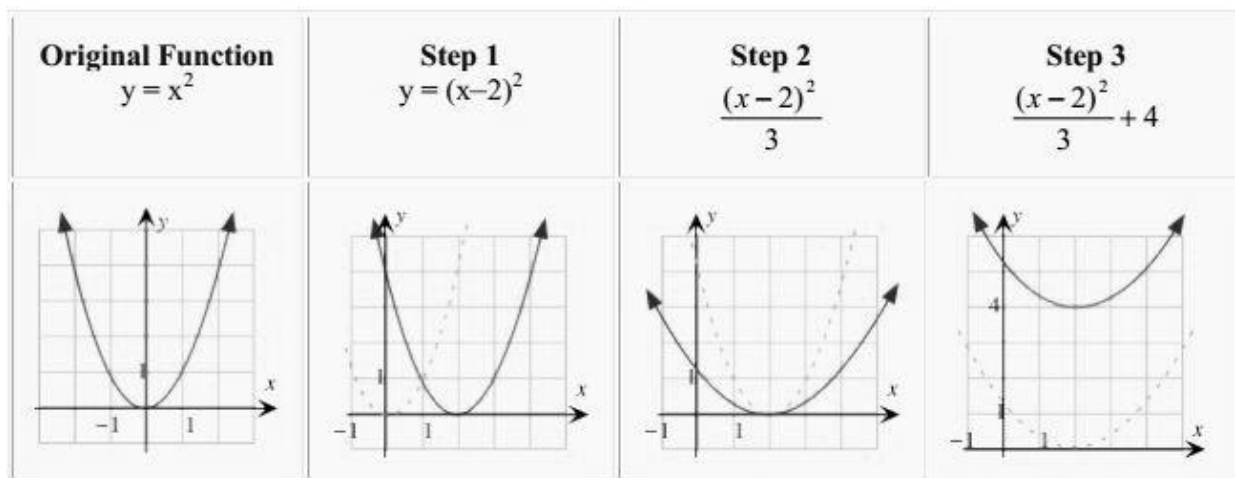
| To graph | Draw the graph of f and: | Changes in the equation of $y = f(x)$ |
|---|--|---|
| Vertical Shifts $y = f(x) + c$ $y = f(x) - c$ | Raise the graph of f by c units Lower the graph of f by c units | C is added to f (x) C is subtracted from f (x) |

Graphing Standard Function & Transformations

| | | |
|---|---|--|
| Horizontal Shifts $y = f(x + c)$ $y = f(x - c)$ | Shift the graph of f to the left c units Shift the graph of f to the right c units | x is replaced with $x + c$ x is replaced with $x - c$ |
| Reflection about the x axis $y = -f(x)$ | Reflects the graph of f about the x axis | $f(x)$ is multiplied by -1 |
| Reflection about the y axis $y = f(-x)$ | Reflect the graph of f about the y axis | x is replaced with $-x$ |

Sample Question:

Sketch the curve for $g(x) = \frac{(x-2)^2}{3} + 4$



Solve for yourself:

Graphing Standard Function & Transformations

1. $f(x) = \sqrt{(x-5)^3}$

2. $g(t) = \frac{1}{5}|3t|$

3. $r(a) = \frac{2}{(3a+4)}$