Module 1.A: Number systems, Order of operations, and Applications

Learning Objectives
1. Classify a number into the correct number system.
2. Order numbers from smallest to largest.
3. Perform a calculation using order of operations.
4. Perform basic operations with fractions.
5. Solve an application word problem involving integers.
6. Apply percents to solve a word problem.

SECTION 1: Classify a number into the correct number system.

Natural or Counting Numbers = \{1, 2, 3, \ldots \} = N
Whole Numbers = \{0, 1, 2, 3, \ldots \} = W
Integers = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \} = Z
   Note that the Natural and Whole Numbers are included in the Integers.

Rational Numbers or Fractions = \left\{ \frac{p}{q} \right\} p and q are integers, q \neq 0 = Q
   Note that the Integers are included in the Rational Numbers, since for example \( \frac{13}{1} \).

Similarly all numbers with terminating or repeating decimals are Rational Numbers:
\[
2.0625 = 2 \frac{1}{16} = \frac{33}{16} \\
0.\overline{6} = 0.66666... = \frac{2}{3} \\
1.7\overline{162} = 1.7162162162... = \frac{127}{74}
\]

Irrational Numbers is the set of non-repeating (also non-terminating) decimals, like \( \pi = 3.1415926535... \) or \( \sqrt{2} = 1.4142135623... \).

Real Numbers (including all of the above) = R

Inclusion Diagram:
EXERCISE 1

Classify the numbers as members of the following set or sets:
A. Natural Numbers   B. Whole Numbers   C. Integers   D. Rationals   E. Irrationals

\( \pi, -3, \frac{1}{3}, \sqrt{6}, 0, 2, 0.\overline{3}, -\frac{1}{9}, 100, \sqrt{7} \)

SOLUTION

A. \(2, 100\)
B. \(0, 2, 100\)
C. \(-3, 0, 2, 100,\)
D. \(-3, \frac{1}{3}, 0, 2, 0.\overline{3}, -\frac{1}{9}, 100,\)
E. \(\pi, \sqrt{6}, \sqrt{7}\)

SECTION 2  Order the numbers from smallest to largest.

The inequality symbol is a quick means of expressing this order.
We say \(a\) is greater than \(b\) but we can write either \(b < a\) or \(a > b\).
The inequality symbol always points to the smaller quantity:

Larger Number > Smaller Number
Smaller Number < Larger Number

<table>
<thead>
<tr>
<th>Inequality Symbol</th>
<th>Verbal Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a &gt; b)</td>
<td>(a) is greater than (b)</td>
<td>(-2 &gt; -3)</td>
</tr>
<tr>
<td>(a &lt; b)</td>
<td>(a) is less than (b)</td>
<td>(-4 &lt; 2)</td>
</tr>
<tr>
<td>(a \geq b)</td>
<td>(a) is greater than or equal to (b)</td>
<td>(3 \geq 3)</td>
</tr>
<tr>
<td>(a \leq b)</td>
<td>(a) is less than or equal to (b)</td>
<td>(4 \leq 4)</td>
</tr>
</tbody>
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EXERCISE 2

Order the numbers from smallest to largest. \(-3, \frac{-2}{3}, \frac{1}{4}, -\frac{3}{2}, 2, \frac{1}{6}, \frac{2}{7}\)

SOLUTION

\(-3, \frac{-3}{2}, \frac{-2}{3}, \frac{1}{6}, \frac{1}{4}, \frac{2}{7}, 2\) since \(-3 < -\frac{3}{2} < -\frac{2}{3} < \frac{1}{6} < \frac{1}{4} < \frac{2}{7} < 2\)
1. Classify the numbers as members of the integers.

   (i) $\pi$, (ii) $-\frac{1}{4}$, (iii) $\sqrt{5}$, (iv) 0, (v) 0.1

A (i) and (iv) only       B all except (i)       C (iii) and (iv) only
D (iv) only               E I do not know

2. Classify the numbers as members of the rational numbers.

   (i) $\pi$, (ii) $-\frac{1}{4}$, (iii) $\sqrt{5}$, (iv) 0, (v) 0.1

A all except (iii)       B all except (i) and (iii)       C (ii) and (iv) only
D (iv) only               E I do not know

3. Order the numbers from smallest to largest. $\frac{-1}{4}, -2, \frac{1}{5}, \frac{3}{4}, -2, \frac{1}{3}$

A $\frac{-1}{4}, \frac{-2}{5}, -2, \frac{1}{3}, \frac{3}{4}, 2$       B $-2, \frac{-2}{5}, -1, \frac{1}{4}, \frac{3}{4}, 2$

C $-2, \frac{-2}{5}, -1, \frac{3}{4}, \frac{1}{3}, 2$       D $-2, \frac{-1}{4}, \frac{-2}{5}, \frac{1}{3}, \frac{3}{4}, 2$

E I do not know