Module 3.B: Algebra - Solving Equations

SECTION 1: Solve a linear equation.

Steps to solving a linear equation.
- Use the distributive property to remove all parentheses.
- Collect like terms.
- Isolate the variable on one side of the equal sign, numbers to the other side of the equal sign.

EXERCISE 3

Solve $4x - (x - 8) + 2x - 3(x + 1) = 1$

SOLUTION

$4x - (x - 8) + 2x - 3(x + 1) = 1$
$4x - x + 8 + 2x - 3x - 3 = 1$
$2x + 5 = 1$ $\Rightarrow$ $2x = -4$ $\Rightarrow$ $x = -2$

SECTION 2: Solve a system of linear equations.

EXERCISE 4

Solve the following system of equations.

$$\begin{cases} 4x - 3y = 1 \\ -6x + 6y = -1 \end{cases}$$
SOLUTION

To solve, we will use the addition method. We will multiply equation 1 by 2 to eliminate the variable $y$.

\[
\begin{align*}
2(4x - 3y &= 1) &\Rightarrow 8x - 6y = 2 \\
-6x + 6y &= -1
\end{align*}
\]

\[
8x - 6y = 2
\]

\[
-6x + 6y = -1
\]

Now, we will eliminate the variable $y$.

\[
2x = 1 \Rightarrow x = \frac{1}{2}
\]

Now, we substitute $x = \frac{1}{2}$ into the one of the equations.

\[
4\left(\frac{1}{2}\right) - 3y = 1 \Rightarrow 2 - 3y = 1 \Rightarrow -3y = 1 - 2 \Rightarrow -3y = -1 \Rightarrow y = \frac{-1}{-3} \Rightarrow y = \frac{1}{3}
\]

The solution is $\left(\frac{1}{2}, \frac{1}{3}\right)$.

SECTION 3 : Solve a quadratic equation.

Steps to solve a quadratic equation.
First: collect all terms on one side of the equation.
Then:
- Factor (if possible).
- Set each factor equal to zero.
- Solve for $x$.
Or
- Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

EXERCISE 5

Solve $x^2 - 3x = 18$

SOLUTION

\[
x^2 - 3x = 18 \Rightarrow x^2 - 3x - 18 = 0 \Rightarrow (x - 6)(x + 3) = 0 \Rightarrow x - 6 = 0 \text{ or } x + 3 = 0 \Rightarrow x = 6 \text{ or } x = -3
\]
SECTION 4: Solve an application word problem.

EXERCISE 6

You earn a 75, 87, 58 and a 94 on the first four exams respectively. What do you need to earn on the last exam to earn a B for the course (an 80 percent average)? Note all of the exams are weighted the same.

SOLUTION

To get a B, you need \((80)(5) = 400\) points.
So far, you earned \(75 + 87 + 58 + 94 = 314\).
\(400 – 314 = 86\). You need to earn an 86 on the last exam.
MODULE 3.B - ASSESSMENT

3. Solve \(2x - (3x - 8) + 5x - 6(x - 1) = 11\)

A \(-\frac{3}{2}\)  B \(\frac{3}{2}\)  C \(-\frac{25}{2}\)  D 1  E I do not know

4. Solve the following system of equations, \(\begin{align*}
3x - 2y & = 6 \\
-2x + 4y & = -8
\end{align*}\)

A \(\left(1, \frac{3}{2}\right)\)  B \(\left(1, -\frac{3}{2}\right)\)  C \(\left(-\frac{1}{2}, -\frac{9}{4}\right)\)  D \(\left(\frac{5}{2}, \frac{3}{4}\right)\)  E I do not know

5. You earn a 60, 97, 80 and a 85 on four exams. What do you need to earn on the last exam to earn a B for the course (an 80 percent average)? All of the exams are weighted the same.

A 70  B 68  C 70.5  D none of these  E I do not know

6. Solve \(x^2 - 5x - 36 = 0\)

A \(x = 9, -4\)  B \(x = -9, 4\)  C \(x = 6, 1\)  D \(x = 9, 4\)  E I do not know