

Module 5A : Exponents, Logarithms, and Miscellaneous Topics

Learning Objectives:

1. Simplify using the rules of exponents.
2. Simplify terms with rational exponents.
3. Rationalize a denominator.
4. Simplify a logarithm using the definition of logs.
5. Expand and contract logarithm expressions.
6. Solve an equation in quadratic form.
7. Solve exponential equations.
8. Solve radical equations.

SECTION 1 : Simplify using the rules of exponents.

Rules of Exponents: Assume all bases are positive.

- $x^{-n} = \frac{1}{x^n}$.
- $x^a x^b = x^{a+b}$. Example: $2^3 2^2 = 2^{3+2} = 2^5$.
- $\frac{x^a}{x^b} = x^{a-b}$. Example: $\frac{2^3}{2^2} = 2^{3-2} = 2^1$.
- $(x^a)^b = x^{ab}$. Example: $(2^2)^3 = 2^{2(3)} = 2^6$.
- $x^0 = 1$. Example: $(2)^0 = 1$.
- $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$, where $x, y \neq 0$. Example: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

EXERCISE 1

Simplify: a) $(3b)^0$

b) $3b^0$

SOLUTION:

a) $(3b)^0 = 1$

b) $3b^0 = 3(1) = 3$

EXERCISE 2

Simplify: a) $-ab^{-2}a^3(b^2)^{-1}$

b) $a^{-4}b^5(2a^{-1}b^2)^{-3}$

SOLUTION:

$$\text{a) } -ab^{-2}a^3(b^2)^{-1} = -aa^3b^{-2}b^{-2} = -a^{3+1}b^{-2-2} = -a^4b^{-4} = \frac{-a^4}{b^4}$$

b)

$$\begin{aligned} & a^{-4}b^5(2a^{-1}b^2)^{-3} \\ &= a^{-4}b^5(2)^{-3}(a^{-1})^{-3}(b^2)^{-3} \\ &= a^{-4}a^{-1(-3)}b^5b^{2(-3)}2^{-3} \\ &= a^{-4}a^3b^5b^{-6}2^{-3} = a^{-1}b^{-1}2^{-3} = \frac{1}{2^3 ab} \end{aligned}$$

MODULE 5A - ASSESSMENT

_____1. Simplify $-(2c)^0$

A $-2c$ **B** -1 **C** 1 **D** $2c$ **E** I do not know

_____2. Simplify $-b^2a^{-2}b^{-5}$

A $a^{-2}b^{-3}$ **B** $-\frac{a^2}{b^3}$ **C** $-\frac{1}{a^2b^3}$ **D** none of these **E** I do not know

_____3. Simplify $a^{-3}(a^3b^{-2})^{-4}$

A $\frac{b^8}{a^{15}}$ **B** $\frac{b^8}{a^{-15}}$ **C** b^8 **D** none of these **E** I do not know