

Module 5C : Exponents, Logarithms, and Miscellaneous Topics
SECTION 1 : Simplify a logarithm using the definition of logs.

If $b^c = d \Rightarrow \log_b d = c$

EXERCISE 5

Simplify a) $\log_5\left(\frac{1}{125}\right)$ b) $\log_2(32)$ c) $\log_{\frac{1}{2}}(8)$

SOLUTION

a) $\log_5\left(\frac{1}{125}\right) = x \Rightarrow 5^x = \frac{1}{125} \Rightarrow 5^x = 5^{-3} \Rightarrow x = -3$

b) $\log_2(32) = x \Rightarrow 2^x = 32 \Rightarrow 2^x = 2^5 \Rightarrow x = 5$

c) $\log_{\frac{1}{2}}(8) = x \Rightarrow \left(\frac{1}{2}\right)^x = 8 \Rightarrow (2^{-1})^x = 2^3 \Rightarrow -x = 3 \Rightarrow x = -3$

SECTION 2 : Expand and contract logarithm expressions.

Rules of Logarithms: assume all bases are positive.

- $\log_a(xy) = \log_a(x) + \log_a(y)$
- $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
- $\log_a(x^y) = y\log_a(x)$

EXERCISE 6

a) Expand $\log_b \frac{x^3 y^5}{6z^2}$

b) Simplify $4\log_b x + 2\log_b y - 3\log_b z$

SOLUTION

a) Expand

$$\begin{aligned}\log_b \frac{x^3 y^5}{6z^2} &= \log_b(x^3 y^5) - \log_b(6z^2) \\ &= \log_b x^3 + \log_b y^5 - (\log_b(2 \cdot 3) + \log_b z^2) \\ &= 3\log_b x + 5\log_b y - \log_b 2 - \log_b 3 - 2\log_b z\end{aligned}$$

b) Simplify

$$\begin{aligned}4\log_b x + 2\log_b y - 3\log_b z &= \log_b x^4 + \log_b y^2 - \log_b z^3 \\ &= \log_b \frac{x^4 y^2}{z^3}\end{aligned}$$

MODULE 5C - ASSESSMENT

_____6. Simplify $\log_4(64)$

- A** 8 **B** 4 **C** 3 **D** 6 **E** I do not know

_____7. Expand: $\log_b \frac{x^6 y^9}{4z}$

A $6\log_b x + 9\log_b y - \log_b 4z$

C $\log_b 6x + \log_b 9y - \log_b 4z$

E I do not know

B $6\log_b x + 9\log_b y - \log_b 4 + \log_b z$

D $\log_b(6x \cdot 9y) - \log_b 4z$